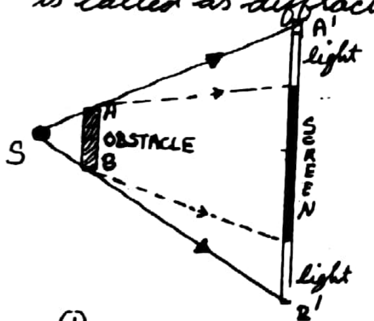
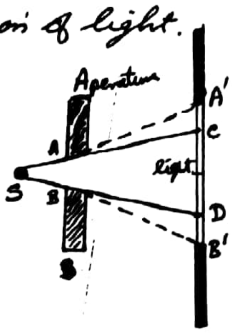
DIFFRACTION

The phenomenon of bending of light around corners of an obstacle or an aperature into the region of Geometrical shadow of obstacle is called as diffraction of light.





Exp (1) The diffraction of wave is more pronounced if the size of the obstacle / aperature is of the order of the wavelength of the wave.

(2) Minimum distance of the observer from obstacle in order to observe the diffraction of light of wavelength λ is $x = \frac{d^2}{4\lambda}$.

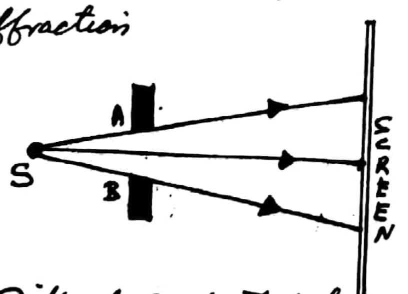
Types of Diffraction

It is of two types

1. Fresnel and 2. Fraunhoffer

1) Fresnel Diffraction

In this case both the source or screen or both at finite distance from the obstacle / aperature causing diffraction

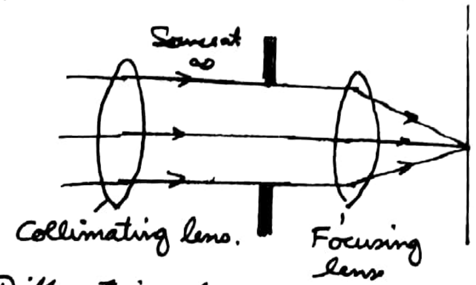


e.g. Diffraction at straight edge, Small opaque disc, Narrow wire

2) Fraunhoffer Diffraction

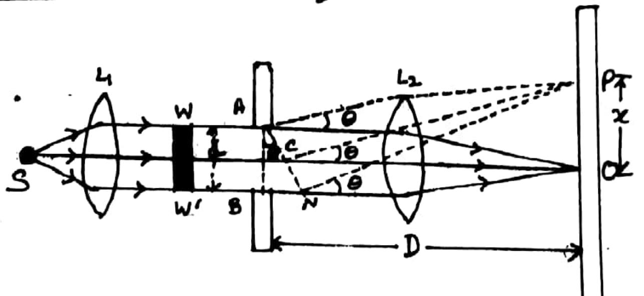
In this case both the source and the screen are at infinite distance from the obstacle / aperature causing diffraction.

In this case lenses are required to render light parallel before falling on the obstacle / aperature and to focus the diffraction pattern on to the screen after diffraction has occured.



eg. Diffraction at single slit, double slit and diffraction grating.

Diffraction at Single slit



Consider a monochromatic light source S. Light from S is made parallel after refraction through lens L_1, thereby forming a plane wavefront WW' which is incident on a slit AB of width 'd'. As per Hygens principle, each point of slit AB acts as a source of secondary disturbances / wavelets.

Consider point O on the screen placed at a distance D from the obstacle (slit) AB. At pt O constructive interference takes place as all the wavelets from AB reaches pt O in phase. [$\because AO = BO$]

Thus point O forms "Central Maxima"

If light is diffracted through an angle θ so will be the secondary wavelets as shown in figure. Let these wavelets meet the screen at point P. The point P will be maxima or minima depends upon the Path Difference b/w the secondary wavelets originating from the corresponding points of the wavefronts.

PATH DIFFERENCE

In order to determine the path difference b/w secondary wavelets originating from corresponding points A & B. of the plane wavefront. Draw AN⊥BP. Then path difference is BN

From ΔBAN $\sin\theta = \frac{BN}{AB}$

$BN = AB \sin\theta$

Path difference $= BN = d \sin\theta$.

Max/Min. of a pt. on basis of Path diff.

a) If path difference is equal to wavelength i.e $BN = d\sin\theta = \lambda$

Then pt P is a minima. Reason

If we divide the wavefront into two equal ~~half~~ halves AC & CB. s.t the path difference is $\lambda/2$ meaning that these wavelets meet at pt P out of phase. (as phase $= \pi$)

Similarly path difference b/w wavelets originating from B & C is also $\lambda/2$ or phase π resulting in destructive superposition at pt P.

Thus for 1st minima

$d \sin\theta_1 = \lambda$.

$\sin\theta_1 = \frac{\lambda}{d} \Rightarrow \theta_1 = \frac{\lambda}{d}$

b) Similarly if $BN = 2\lambda$ then slit AB is imagined to be split into FOUR equal halves. The path difference b/w the secondary wavelets originating from the corresponding points of each half $= (2\lambda/4) = \lambda/2$. Thus path difference for Second Minima

$d \sin\theta_2 = 2\lambda$

$\sin\theta_2 = \frac{2\lambda}{d} \Rightarrow \theta_1 = \frac{2\lambda}{d}$

Thus in general for minima

$$\boxed{\theta_n = \frac{n\lambda}{d} \quad \text{——(i)} \quad \text{for } n = 1,2,3 \ldots \text{an integer}}$$

c) For Secondary Maxima.

If the path difference $BN = d\sin\theta$ is an odd multiple of $\lambda/2$. then constructive interference takes place at point P. i.e wave front is split into odd no. of equal parts.....

or $d\sin\theta_n = (2n+1)\lambda/2$.

$$\boxed{\theta_n = (2n+1)\lambda/2 . \quad \text{——(ii)} \quad \text{for } n = 1,2,3 \ldots}$$

Results

Condition for Minima

The path difference is $n\lambda$ or $(2n)\frac{\lambda}{2}$ & direction of nth minima

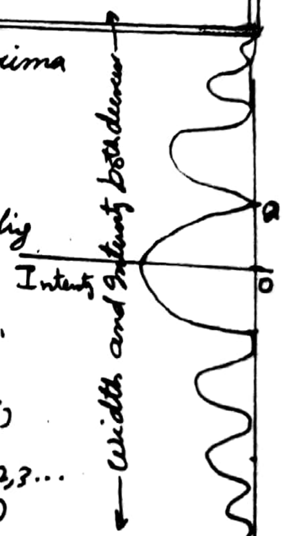
$$\theta_n = \frac{n\lambda}{d}.$$

Condition for Maxima

The path difference is $(2n+1)\lambda/2$ & direction for nth maxima

$$\theta_n = \frac{(2n+1)\lambda/2}{d}.$$

Width of Central Maxima



let $OQ = y$. s.t for nth order minima $PQ = y_n$ and corresponding angle of diffraction is θ_n

∴ Path difference

$BN = d\sin\theta_n = n\lambda$ —(i)

$\sin\theta_n = n\frac{\lambda}{d}$ for $n = 1,2,3\ldots$ —(ii)

If f is the focal length of the lens and it is placed close to the slit then

$\sin\theta_n \simeq \theta_n = \frac{y_n}{f}$ —(iii)

From eq's (ii) & (iii)

$\frac{y_n}{f} = n\frac{\lambda}{d}$

$y_n = n\frac{\lambda}{d}f$ —(iv)

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as lens is close to slit $\therefore f \simeq D$

$$y_n = n\frac{\lambda}{d}D \quad \text{—(v)}$$

Similarly for nth order maxima its distance from centre O is

$$y_n = (2n+1)\frac{\lambda D}{d} \quad \text{—(vi)}$$

Now as the central maxima is surrounded by two first order minima on either side here.

$$\begin{pmatrix}\text{Width of}\\ \text{Central}\\ \text{Maxima}\end{pmatrix} = 2 \times \begin{pmatrix}\text{Distance of Ist order}\\ \text{minima from pt O}\end{pmatrix}$$

$$\beta_0 = 2(y_1) = 2\frac{\lambda D}{d}.$$

$$\boxed{\beta_0 = \frac{2\lambda D}{d}}$$

Thus i) $\beta_0 \propto \lambda$
hence width ~~of~~ central maxima is small for Violet colour and large for Red colour

ii) $\beta_0 \propto \frac{1}{d}$
Thus if width of slit is large central max is small and vice versa.

Intensity

As we move away from the Central maxima, the intensity of secondary maximas decreases rapidly as

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \text{ where } \alpha = \frac{\pi d \sin\theta}{\lambda}.$$

for $\theta = 0°$ $\lim_{\alpha\to 0}\left(\frac{\sin\alpha}{\alpha}\right) = 1 \therefore I = A^2 = I_0.$

Resolving Power

Diffraction of light limits the ~~the~~ ability of optical instruments to form clear images of objects when they are close to each other.

Limit of Resolution: It is defined as the minimum distance of seperation b/w two points so that they can be seen as seperate (or just resolved) by the optical instrument.

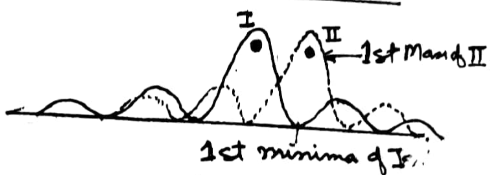
#3

Resolving Power: The ability of an optical instrument to form distinct seperate images of the two closely placed points or objects is called as its Resolving Power.

Thus.

$$\text{Resolving Power (R.P)} \propto \frac{1}{(\text{limit of resolution})}$$

Rayleigh Criteria of Resolution



Two very close lying objects are said to be just resolved if the 1st Maxima of IInd object falls on the 1st Minima of Ist object or viceversa at the observers eye.

Comparison b/w Interference & Diffraction

1. Interference is due to superposition of wave-fronts from two Coherent sources.
Diffraction is due to superpostion of wavelets originating from different pts on same wavefront.
2. In Interfernce all maximas are of equal intensity
Bright fringes are of varing intensity with Central maxima as brightest
3. Dark fringes are perfectly dark.
In it dark fringes are not perfectly dark
4. In interference β (fringewidth of both bright and dark) are equal.
In Diffraction β_0 is largest and it decreases as we move away from the Central maxima
5. These are large in number
Diffraction pattern has few fringes
6. In interference bands are equally spaced.
In Diffraction bands are unequally spaced.

Numericals Diffraction

1. A slit of width 'a' is illuminated by light of wavelength 6000Å. For what value of 'a' will the (i) First max. fall at an angle of diffraction of 30°? (ii) First minimum fall at an angle of diffraction of 30°.

Hint i) $\sin\theta_n = \frac{(2n+1)}{a}\frac{\lambda}{2}$ $n=1$ $\theta_1 = 30°$

$a = \frac{3}{2}\frac{\lambda}{\sin 30°}$ mts. 1.8×10^{-6} mts

(ii) $\sin\theta_n = \frac{n\lambda}{a}$ .... 1.2×10^{-6} m.

2. Red light of wavelength 6500Å from a distant source falls on a slit 0.50mm wide. What is the distance b/w the two dark bands on each side of the central bright band of diffraction pattern observed on a screen 1.8m from the slit.

Hint Distance b/w two dark bands on each side of max = Width of maxima

$x = \frac{2\lambda D}{d}$ 4.68mm.

3. Light of wavelength 5×10^{-7} m is diffracted by an aperture of width 2×10^{-3} m. For what distance travelled by the diffracted beam does the spreading due to diffraction become greater than the width of the aperture.

Hint $Z_F = d^2/\lambda$.

4. calcuate the distance a beam of light of wavelength 500nm can travel without significant broadning, if aperture is 3mm wide.

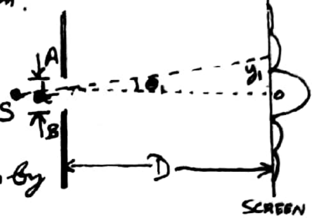
Hint $Z_F = d^2/\lambda$.

Left over article

Fresnel Distance

It is defined as the distance of the screen from the slit where the spreading of light due to diffraction from the centre of the screen is equal to the size (width) of the slit.

Derivation:- Now the diffraction pattern of a single slit consists of secondary maxima and minima surrounding the central maxima. The spread of light on the screen depends upon the size of the Central Maximum.



Angular position of first minima on either side of Central max. is given by

$\sin\theta_1 = \frac{\lambda}{d}$.

$\theta_1 \simeq \frac{\lambda}{d}$ [as θ_1 is small] —(i)

This is called as Half Angular Width of the central maxima.

Let D be the distance b/w screen & slit. The linear spread of central maxima is given by $y_1 = D\theta_1 = \frac{\lambda D}{d}$.

here $D = Z_F$ = Fresnel Distance ; $y_1 = d$

$\therefore d = \frac{\lambda Z_F}{d}$ or $Z_F = \frac{d^2}{\lambda}$.

Thus Fresnel distance depends upon i) Size of slit ii) Wavelength of light.

Numericals cont:-

5. Determine Fresnel distance for an aperture of 1mm for a $\lambda = 1000$nm. (6.183)

6. (6.182) Determine angular seperation b/w central maximum and the first order maximum of the diffraction pattern due to a single slit of width 0.25mm. when light of wavelength 6890Å is incident normally on it.

Hint $\theta_n = \frac{(2n+1)}{d}\frac{\lambda}{2}$ $n=1$ (3.5×10^{-3} rad)

7. (6.182) Determine angular direction of third minimum in a diffraction pattern produced by light of 5000Å passing through a slit of width 0.1mm.

Hint $\theta_n = \frac{n\lambda}{d}$.   ($1.5\times10^{-3}$ rad)